OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Mechanics 2

4729

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 9 January 2009
Morning

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

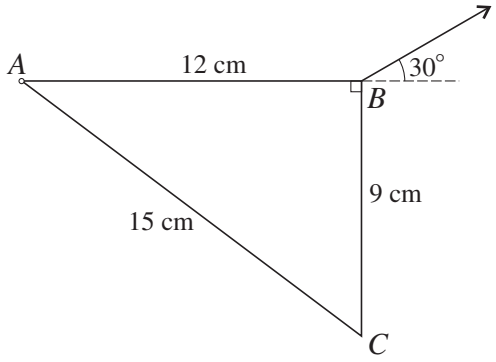
- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

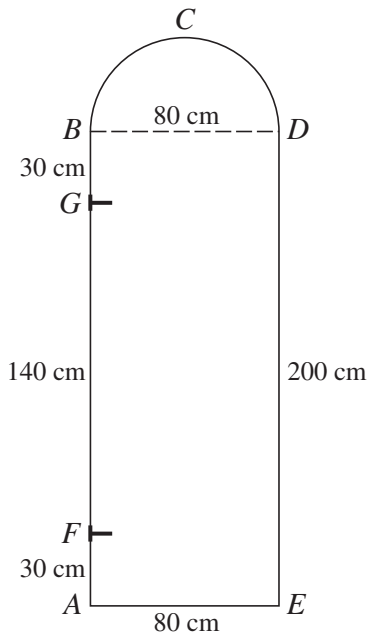
1 A stone is projected from a point on level ground with speed $20\,\text{m}\,\text{s}^{-1}$ at an angle of elevation of $\theta°$ above the horizontal. When the stone is at its greatest height it just passes over the top of a tree that is 17 m high. Calculate θ. **[4]**

2



A uniform right-angled triangular lamina ABC with sides $AB = 12$ cm, $BC = 9$ cm and $AC = 15$ cm is freely suspended from a hinge at its vertex A. The lamina has mass 2 kg and is held in equilibrium with AB horizontal by means of a string attached to B. The string is at an angle of 30° to the horizontal (see diagram). Calculate the tension in the string. **[4]**

3



A door is modelled as a lamina $ABCDE$ consisting of a uniform rectangular section $ABDE$ of weight 60 N and a uniform semicircular section BCD of weight 10 N and radius 40 cm. AB is 200 cm and AE is 80 cm. The door is freely hinged at F and G, where G is 30 cm below B and F is 30 cm above A (see diagram).

(i) Find the magnitudes and directions of the horizontal components of the forces on the door at each of F and G. **[4]**

(ii) Calculate the distance from AE to the centre of mass of the door. **[6]**

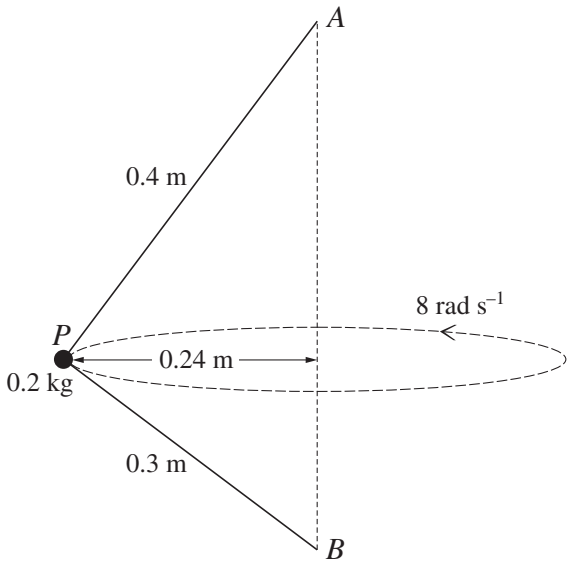
4 A car of mass 800 kg experiences a resistance of magnitude kv^2 N, where k is a constant and v m s^{-1} is the car's speed. The car's engine is working at a constant rate of P W. At an instant when the car is travelling on a horizontal road with speed 20 m s^{-1} its acceleration is 0.75 m s^{-2}. At an instant when the car is ascending a hill of constant slope 12° to the horizontal with speed 10 m s^{-1} its acceleration is 0.25 m s^{-2}.

(i) Show that $k = 0.900$, correct to 3 decimal places, and find P. **[7]**

The power is increased to $1.5P$ W.

(ii) Calculate the maximum steady speed of the car on a horizontal road. **[3]**

5



A particle P of mass 0.2 kg is attached to one end of each of two light inextensible strings, one of length 0.4 m and one of length 0.3 m. The other end of the longer string is attached to a fixed point A, and the other end of the shorter string is attached to a fixed point B, which is vertically below A. The particle moves in a horizontal circle of radius 0.24 m at a constant angular speed of 8 rad s^{-1} (see diagram). Both strings are taut, the tension in AP is S N and the tension in BP is T N.

(i) By resolving vertically, show that $4S = 3T + 9.8$. **[4]**

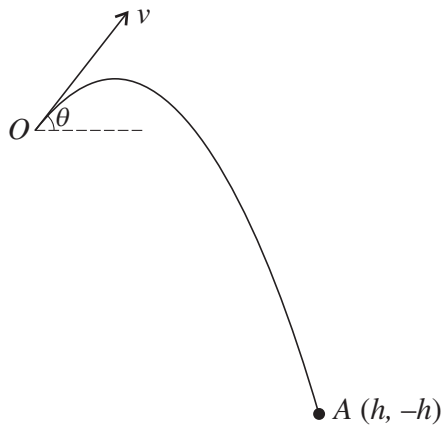
(ii) Find another equation connecting S and T and hence calculate the tensions, correct to 1 decimal place. **[8]**

[Questions 6 and 7 are printed overleaf.]

6 A particle is projected from a point O with speed v m s^{-1} at an angle of elevation θ above the horizontal and it moves freely under gravity. The horizontal and upward vertical displacements of the particle from O at any subsequent time, t seconds, are x m and y m respectively.

(i) Express x and y in terms of θ and t, and hence show that

$$y = x\tan\theta - \frac{4.9x^2}{v^2\cos^2\theta}.$$ **[4]**



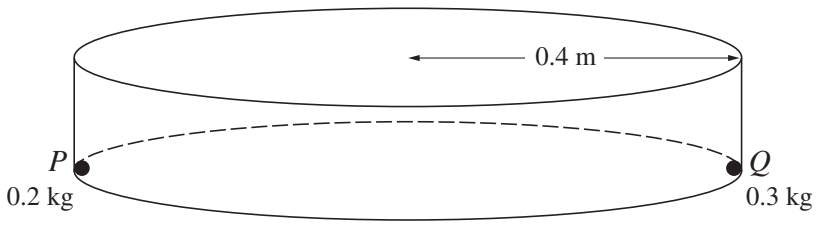
The particle subsequently passes through the point A with coordinates $(h, -h)$ as shown in the diagram. It is given that $v = 14$ and $\theta = 30^\circ$.

(ii) Calculate h. **[4]**

(iii) Calculate the direction of motion of the particle at A. **[5]**

(iv) Calculate the speed of the particle at A. **[2]**

7



Two small spheres, P and Q, are free to move on the inside of a smooth hollow cylinder, in such a way that they remain in contact with both the curved surface and the base of the cylinder. The mass of P is 0.2 kg, the mass of Q is 0.3 kg and the radius of the cylinder is 0.4 m. P and Q are stationary at opposite ends of a diameter of the base of the cylinder (see diagram). The coefficient of restitution between P and Q is 0.5. P is given an impulse of magnitude 0.8 N s in a tangential direction.

(i) Calculate the speeds of the particles after P's first impact with Q. **[8]**

Q subsequently catches up with P and there is a second impact.

(ii) Calculate the speeds of the particles after this second impact. **[7]**

(iii) Calculate the magnitude of the force exerted on Q by the curved surface of the cylinder after the second impact. **[2]**

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1	$(20 \sin\theta)^2 = 2 \times 9.8 \times 17$	M1	or B2 for max ht = $v^2\sin^2\theta/2g$
		A1	
	$\sin\theta = \sqrt{(2\times9.8\times17)} \div 20$	M1	subst. values in above
	$\theta = 65.9°$	A1 **4**	**4**

2	$\bar{x} = 8$	B1	
	T sin30° x 12 = 8 x 2 x 9.8	M1	ok if g omitted
		A1 ft	ft their $\bar{x}$
	T = 26.1	A1 **4**	**4**

3 (i)	140 x X = 40 x 70	M1	
	X = 20 N	A1	
	at *F* 20 N to the right	B1	inspect diagram
	at *G* 20 N to the left	B1 **4**	**SR** B1 for correct directions only
(ii)	đ = (2x40sinΠ/2)÷3Π/2	M1	must be radians
		A1	
	đ = 17.0	A1	16.98 160/3Π (8/15Π m)
	$70\bar{y} = 100\times60 + 217 \times 10$	M1	
		A1 ft	ft 200 + their đ or 2 + their đ (m)
	$\bar{y} = 117$	A1 **6**	116.7 **10**

4 (i)	$P/10 - 800\times9.8\sin12° - 100k = 800\times0.25$	M1	$P/10 = D_1$ ok
		A1	D_1 ok
	$P/20 - 400k = 800\times0.75$	M1	$P/20 = D_2$ ok
		A1	$D_1 = 2D_2$ needed for this A1
	solving above	M1	
	$k = 0.900$	A1	**AG** 0.9000395
	$P = 19\ 200$	A1 **7**	or 19.2 kW (maybe in part (ii))
(ii)	$0.9v^2 = 28\ 800/v$	M1	ok if $19200/v$
	solving above	M1 *	**($v^3 = 32\ 000$)**
	$v = 31.7$ m s^{-1}	A1 **3**	**10**

5 (i)	0.8 *S*	B1	vert comp of *S*
	0.6 *T*	B1	vert comp of *T*
	$S\cos\alpha = T\cos\beta + 0.2 \times 9.8$	M1	
	$0.8S = 0.6T + 1.96$ aef	A1 **4**	**AG** $4S = 3T + 9.8$
(ii)	0.6 *S*	B1	
	0.8 *T*	B1	
	$0.2 \times 0.24 \times 8^2$	B1	3.072 384/125
	$S\sin\alpha + T\sin\beta = 0.2 \times 0.24 \times 8^2$	M1	must be $mr\omega^2$
	$6S + 8T = 30.72$	A1	aef
	eliminate *S* or *T*	M1	
	S = 3.4 N	A1	3.411
	T = 1.3 N	A1 **8**	1.282 **12**

6 (i)	$x = v\cos\theta\, t$	B1	
	$y = v\sin\theta\, t - \frac{1}{2} \times 9.8\, t^2$	B1	or g
	substitute $t = x/v\cos\theta$	M1	
	$y = x\tan\theta - 4.9x^2/v^2\cos^2\theta$	A1 **4**	**AG**
(ii)	Sub $y = -h$, $x = h$, $v = 14$, $\theta = 30$	M1	signs must be correct
	$-h = h/\sqrt{3} - h^2/30$	A1	aef
	solving above	M1	
	$h = 47.3$	A1 **4**	
(iii)	$v_v^2 = (14\sin30°)^2 - 2\times9.8\times(-47.3)$	M1	$14\cos30°\ t = 47.3$ ft & $v_v = 14\sin30° - 9.8t$
	(double negative needed) ft their -47.3	A1 ft	$t = 3.90$ (or $dy/dx = 1/\sqrt{3} - x/15$ etc ft)
	$v_v = \pm31.2$	A1	$v_v = \pm31.2$ ($\tan\alpha = 1/\sqrt{3} - 47.3/15$)
	$\tan^{-1}(31.2/14\cos30°)$	M1	$\tan^{-1}(31.2/14\cos30°)$
	$\alpha = 68.8°$ below horiz/21.2° to d'vert.	A1 **5**	68.8°/…..
(iv)	$\frac{1}{2}mx14^2 + mx9.8x47.3 = \frac{1}{2}mv^2$	M1	ft $(12.1^2 + 31.2^2)$
	$v = 33.5$	A1 **2**	33.5 **15**

7 (i)	$p = 4$ m s^{-1}	B1	P's first speed
	$0.8 = 0.2p_1 + 0.3q_1$	M1	
		A1	
	$0.5 = (q_1 - p_1)/4$	M1	
		A1	
	solving above	M1	
	$q_1 = 2.4$ 12/5	A1	Q's first speed
	$p_1 = 0.4$ 2/5	A1 **8**	may be in (ii). **SR** 1 for both negative
(ii)	$0.8 = 0.2p_2 + 0.3q_2$	M1	
		A1	
	$0.5 = (p_2 - q_2)/2$	M1	
		A1	
	solving above	M1	
	$p_2 = 2.2$ 11/5	A1	
	$q_2 = 1.2$ 6/5	A1 **7**	
(iii)	$R = 0.3 \times 1.2^2/0.4$	M1	
	$R = 1.08$ N	A1 **2**	**17**